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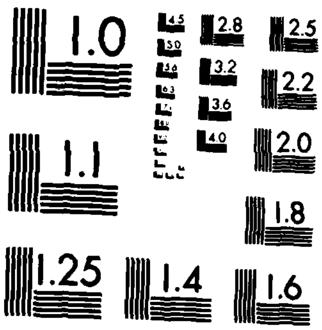

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The Hydrosynclastic Infundibulum

D. L. Book

Laboratory for Computational Physics

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The possibility of setting up a water vortex inside a convergent axisymmetric duct is investigated, with a view toward applications to a proposed high-density-gas-enclosed Z-pinch experiment.

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The Hydrosynclastic Infundibulum*

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In connection with design studies of a high-density-gas-enclosed Z-pinch fusion experiment, Robson (personal communication, 1984) has noted the advantages of surrounding the discharge volume with a water vortex. The latter serves as a first wall, radiation barrier, and heat exhange medium, and scavenges reaction products and unburned fuel. The present note is written in order to show how such a vortex can be set up inside an appropriately shaped convergent axisymmetric duct, or infundibulum.

We consider steady incompressible flows in a geometry indicated schematically in Fig. 1. A jet of water is directed tangentially and slightly downward against the surface of the duct at the input point, where the radius is R_0. The duct axis is assumed to be in the vertical direction. If the inlet flow speed is v_0 and the volume flow rate is $\dot{V}$, then the initial thickness Δ of the layer of water flowing around the duct is given by

$$\dot{V} = 2\pi\ R_0 v_0 \Delta\ \sin\theta_0, \tag{1}$$

where θ_0 is the angle between the direction of the input flow and the tangent to the circle of radius R_0 in the horizontal plane. If $\Delta \ll R_0$, then all the water has the same values of all three components of the velocity vector at the input point. For the time being we assume that the flow is laminar.

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* With apologies to Kurt Vonnegut, Jr.

We can treat steady flows in this configuration using either an Eulerian or Lagrangian representation. Since the flow progresses monotonically downward, either distance traveled along a streamline, its projection on the $r - z$ plane, or the projection of the latter onto the negative z axis (i.e., downward displacement) can serve as a timelike variable parametrizing the motion of a fluid element. Neighboring elements, however, although they remain contiguous, do not travel at the same speed. As a result, any small volume of fluid undergoes shear in the course of its descent through the duct, so that a Lagrangian description based on the use of displacment instead of time is not very convenient. We therefore elect to work in Eulerian coordinates.

The Eulerian equations of steady incompressible axisymmetric laminar flow in cylindrical coordinates are

$$\frac{1}{r}\frac{\partial}{\partial r}(ru) + \frac{\partial w}{\partial z} = 0; \tag{2}$$

$$u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial r} - \frac{v^2}{r} + \frac{1}{\rho}\frac{\partial p}{\partial r} = 0; \tag{3}$$

$$u\frac{\partial v}{\partial r} + w\frac{\partial v}{\partial r} + \frac{uv}{r} = 0; \tag{4}$$

$$u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial r} + g + \frac{1}{\rho}\frac{\partial p}{\partial z} = 0, \tag{5}$$

where we have included the acceleration due to gravity in Eq. (5).

To solve Eqs. (2)-(5), it is useful to introduce curvilinear coordinates. A streamline coordinate σ is defined by

$$d\sigma^2 = dr^2 + dz^2, \tag{6}$$

where

$$\frac{dr}{dz} = \frac{u}{w}. \tag{7}$$

The unit vector in the direction of increasing σ is thus

$$\mathbf{e}_\sigma = \upsilon^{-1}\,(\mathbf{e}_r u + \mathbf{e}_z w), \tag{8}$$

where

$$\upsilon = (u^2 + w^2)^{1/2}. \tag{9}$$

The coordinate transverse to σ is τ; it increases in the direction of the unit vector

$$\mathbf{e}_\tau = \upsilon^{-1}\,(\mathbf{e}_r w - \mathbf{e}_z u). \tag{10}$$

If $\sigma = 0$ at the inlet position, then for fixed τ, σ is the length of a streamline projected on the r-z plane. Likewise, we define $\tau = 0$ at the duct wall, so that for fixed σ, τ measures the distance by which a streamline was separated from the wall at the duct inlet, $0 < \tau < \Delta$.

In terms of σ and τ we can express the radial coordinate of a point on streamline τ by

$$r(\sigma,\tau) = \int_0^\sigma d\sigma' \; u(\sigma',\tau)/\upsilon(\sigma',\tau). \tag{11}$$

Substituting

$$\frac{\partial}{\partial r} = \upsilon^{-1} \left(u \frac{\partial}{\partial\sigma} + w \frac{\partial}{\partial\tau}\right) \tag{12}$$

and

$$\frac{\partial}{\partial z} = \upsilon^{-1} \left(w \frac{\partial}{\partial\sigma} - u \frac{\partial}{\partial\tau}\right) \tag{13}$$

in Eq. (2) we find

$$\frac{\partial\upsilon}{\partial\sigma} + \upsilon \left(\frac{\sin\phi}{r} + \frac{\partial\phi}{\partial\tau}\right) = 0, \tag{14}$$

where $\phi = \tan^{-1}(u/w)$ is the angle a streamline makes with the vertical direction, while Eqs. (3)-(5) become

$$\frac{\partial u}{\partial\sigma} = \frac{v^2}{r\upsilon} - \frac{1}{\rho\upsilon^2} \left(u\frac{\partial p}{\partial\sigma} + w \frac{\partial p}{\partial\tau}\right); \tag{15}$$

$$\frac{\partial v}{\partial\sigma} = - \frac{uv}{r\upsilon}; \tag{16}$$

$$\frac{\partial w}{\partial\sigma} = - \frac{g}{\upsilon} - \frac{1}{\rho\upsilon^2} \left(w \frac{\partial p}{\partial\sigma} - u \frac{\partial p}{\partial\tau}\right). \tag{17}$$

For general inlet conditions and duct geometry, Eqs. (14)-(17) can only be integrated numerically. We can however, learn something about the behavior to be expected in the general case by considering a class of flow fields obtained by making some simple ansatzes regarding the properties of the flow and then choosing a duct shape which conforms with the solutions. To do this we return to the cylindrical form of the system, Eqs. (2)-(5), and write

$$u = -\frac{1}{2} r \frac{\partial w}{\partial z} + \frac{D}{r}; \qquad (18)$$

$$v = C/r; \qquad (19)$$

$$w = w(z); \qquad (20)$$

$$p = \rho\ [A(r) + B(z)], \qquad (21)$$

C and D constant, then Eqs. (2) and (4) are satisfied identically, and we are left with

$$r\ [\frac{1}{4}\ (\frac{dw}{dz})^2 - \frac{1}{2}\ w\ \frac{d^2w}{dz^2}] - \frac{(C^2 + D^2)}{r^3} + \frac{dA}{dr} = 0 \qquad (22)$$

and

$$w\ \frac{dw}{dz} + g + \frac{dB}{dz} = 0. \qquad (23)$$

For Eq. (22) to have a solution it must separate into two equations, one in r and one in z. Thus

$$\frac{1}{4}\ (\frac{dw}{dz})^2 - \frac{1}{2}\ w\ \frac{d^2w}{dz^2} = \text{const} \qquad (24)$$

and

$$\frac{1}{r}\ [\frac{dA}{dr} - \frac{C^2 + D^2}{r^3}] = \text{const}. \qquad (25)$$

The solution of (24) is

$$w(z) = w_0 - \alpha\ (h - z)^2, \qquad (26)$$

w_0, α, and h constant, whence

$$\frac{1}{4}\left(\frac{dw}{dz}\right)^2 - \frac{1}{2} w \frac{d^2w}{dz^2} = \alpha\, w_0 \tag{27}$$

and

$$A(r) = K - \frac{1}{2}\left(\alpha\, w_0 r^2 + \frac{C^2 + D^2}{r^2}\right), \tag{28}$$

K constant. Finally, Eq. (23) yields

$$B(z) = -\ gz - \frac{1}{2} w^2 = -\ gz - \frac{1}{2}\,[w_0 - \alpha\,(h - z)^2]^2. \tag{29}$$

Equation (7), defining the projected streamlines, yields

$$\frac{dr}{dz} = \frac{-\alpha\ r\ (h-z) + D/r}{w_o - \alpha\ (h - z)^2}\ , \tag{30}$$

whose solution is

$$r^2 = \frac{r_0^2\ (w_0 - \alpha\ h^2) + 2Dz}{w_0 - \alpha\ (h-z)^2}\ , \tag{31}$$

r_0 constant. Since the duct surface must be a streamline, which we can specify by means of R_0, its maximum radius at $z = 0$, its radius as a function of vertical position for $z < 0$ is given by

$$R(z) = \left[\ \frac{R_0^2\ (w_0 - \alpha\ h^2) + 2Dz}{w_0 - \alpha\ (h - z)^2}\ \right]^{1/2} \tag{32}$$

Some remarks are in order here regarding the limitations of this solution. From Eq. (26) we see that $w(0) < 0$ implies $w_0 < \alpha h^2$, while $w(z) < 0$ as $z \to -\infty$ implies $\alpha > 0$. To ensure that $R(z) > 0$ for all z, we infer from Eq. (32) that $D > 0$ must hold. In order that the streamlines have positive slope at $z = 0$, Eq. (30) implies that $D < \alpha R_0^2 h$, so that $h > 0$. Hence the streamlines have positive slope for all z. From Eqs. (21) and (29) it is clear that when $-z$ is sufficiently large the pressure becomes negative, which imposes a constraint on the length of the system. And finally, we have no way to ensure that the pressure on the inner surface of the vortex matches that of the gas inside. Generally in experiments this pressure will be roughly independent of z, which cannot be said of the present solution.

To make this solution more meaningful, we assign some more-or-less arbitrary values to the constants appearing in Eqs. (18-(32) and examine the consequences:

Table 1.

Parameter	Value	Units
C	10^4	$cm^2\ s^{-1}$
D	10	$cm^2\ s^{-1}$
Δ	1	cm
h	10^2	cm
K	10^7	dyne cm^{-2}
R_0	10	cm

We give α and w_0 two different sets of values: (1) $\alpha = 10^2\ cm^{-1}\ s^{-1}$, $w_0 = -10^2\ cm\ s^{-1}$, and (2) $\alpha = 5\ 10^{-2}\ cm^{-1}\ s^{-1}$, $w_0 = 10^2\ cm\ s^{-1}$. In Table 2 we show the resulting values of the three velocity components at $z = 0$; the inlet flow speed v_0; the pitch angle θ_0, given by

$$\sin \theta_0 = [u(0)^2 + w(0)^2]^{1/2} \, v_0 \; ; \qquad (33)$$

the duct angle ϕ_0, obtained from (30) using

$$\tan \phi_0 = dR/dz \,\Big|_0 ; \qquad (34)$$

the flow rate $\dot{V}$ given by Eq. (1); the corresponding power $\dot{W} = \frac{1}{2} \dot{V} \rho v^2{}_\theta$ where we set $\rho = 1.0$ g cm^{-3}; the inner and outer radii R_1 and R_2 of the flow (see Fig. 1) at $z = -100$ cm and $z = 200$ cm; and the pressure $p_2(z)$ on the duct surface at $z = 0$, $z_1 = -100$ cm, and $z_2 = -200$ cm.

It is evident that for these choices the dependence on gravity and the radial dependence are both weak; moreover, the layer of water becomes thinner, not thicker, as -z increases, owing to the increase in the vertical velocity component.

Table 2.

	Case (1)	Case (2)
$u(0)$	-9.0 cm s^{-1}	-24 cm s^{-1}
$v(0)$	10^3 cm s^{-1}	10^3 cm s^{-1}
$w(0)$	-200 cm s^{-1}	-400 cm s^{-1}
v_0	1.02×10^3 cm s^{-1}	1.08×10^3 cm s^{-1}
θ_0	11.3 deg	21.8 deg
ϕ_0	2.6 deg	7.1 deg
$\dot{V}$	$12.6\times10^3 cm^3\ s^{-1}$	$25.2\times10^3 cm^3 s^{-1}$
$\dot{W}$	655 W	1.46 kW
$R_1(z_1)$	6.0 cm	4.2 cm
$R_2(z_1)$	6.6 cm	4.7 cm
$R_1(z_2)$	4.5 cm	2.9 cm
$R_2(z_2)$	4.9 cm	3.1 cm
$p_2(0)$	9.5×10^6 dyne cm^{-2}	9.5×10^6 dyne cm^{-2}
$p_2(z_1)$	8.9×10^6 dyne $cm-^{-2}$	7.7×10^6 dyne cm^{-2}
$p_2(z_2)$	7.4×10^6 dyne cm^{-2}	5.0×10^6 dyne cm-2

Now let us consider the effect of restoring the viscous damping terms omitted from Eqs. (3)-(5). We assume that viscous processes are important only in a thin boundary layer next to the surface of the duct, and initially we assume that the flow in the boundary layer is laminar, so that the kinematic or molecular viscosity ν can be used. For flow in pipes this is a good assumption provided that perturbations in the flow at the entrance to the pipe are kept as small as possible (see, e.g., L.D. Landau and E.M. Lifshitz, Fluid Mechanics, Pergamon, London, 1959, §29).

The thickness of a laminar boundary layer is given (Landau and Lifshitz, op. cit., §39) by

$$\delta = G_1(\nu\ S/v)^{1/2}, \tag{35}$$

where G_1 is a geometrical constant of order unity and $S \approx -z$ is the distance traveled along the boundary. [There is some question in my mind whether S should not be the total length of a (helical) streamline.] Taking $G_1 \approx 1$, $S = 100$ cm, $v = 10^3$ cm s^{-1}, and $\nu = 10^{-2}$ poise, we find that $\delta \approx 3\ 10^{-2}$ cm. The frictional force per unit area of the surface of the duct is given by

$$\sigma \approx G_2\rho(\nu\ v^3/S)^{1/2}. \tag{36}$$

The power needed to overcome frictional losses in the boundary layer is

$$\begin{aligned}\dot{W}_f &\approx 2\pi\int R\sigma\ v dz \\ &\approx 2\pi\ \overline{vR\sigma}\ S,\end{aligned} \tag{37}$$

where the bar represents an average. Taking the geometrical factor $G_2 \approx 1$ and setting $R \approx 5$ cm, we find $\dot{W}_f \approx 100$ watts.

The boundary layer may, however, become turbulent as a result of the introduction of disturbances at the inlet, through the action of duct surface irregularities, or because a critical Reynolds number is exceeded

(Landau and Lifshitz, op. cit., §42-44). If that happens, the boundary layer thickness is given by

$$\delta \sim v_* \, S/v, \tag{38}$$

where

$$v = G_3 v_* \log(v_* S/\nu), \tag{39}$$

Since v_* varies slowly as a function of S, δ now increases almost linearly with S, in contrast with (35). The frictional force per unit area in the boundary layer is given by

$$\sigma = \rho v_*^2. \tag{40}$$

Evidently δ grows rapidly, approaching the thickness of the whole vortex in a few centimeters or less. When this happens, the entire flow goes turbulent and boundary layer theory becomes inapplicable. It is obvious in any event that viscous losses must be much larger in the turbulent case than for laminar flow.

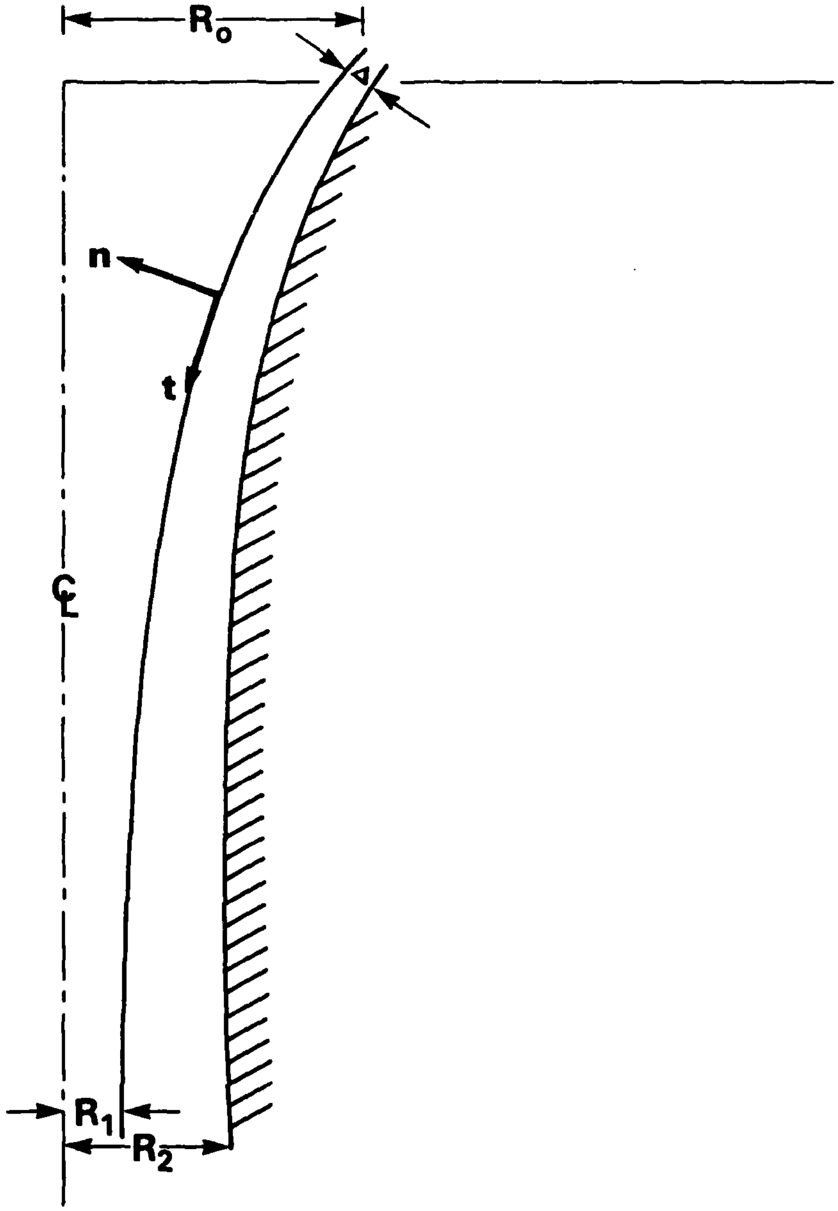

Figure 1

Schematic cross section of water vortex guided by infundibular duct. Initial ($z = 0$) radius of layer is R_0, thickness is Δ. At some finite $z < 0$, inner radius is R_1 and outer radius is R_2. At any point on a streamline, $\underset{\sim}{n} = \underset{\sim}{e}_\tau$ and $\underset{\sim}{t} = \underset{\sim}{e}_\sigma$ represent unit normal and unit tangent, respectively.

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